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Determination of the steady state response of viscoelastically point-supported rectangular specially orthotropic plates with added concentrated masses

T. Kocatürk[a,*], S. Sezer[b], C. Demir[b]

[a] *Faculty of Civil Engineering, Yildiz Technical University, Yildiz-Besiktas, 34349 Istanbul, Turkey*
[b] *Faculty of Mechanical Engineering, Yildiz Technical University, Yildiz-Besiktas, 34349 Istanbul, Turkey*

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Abstract

Vibration of orthotropic rectangular plates having viscoelastic point supports at the corners with symmetrically added four concentrated masses rigidly mounted on the two diagonals of the plate is analyzed. The Lagrange equations are used to examine the steady state response to a sinusoidally varying force applied at the centre of a viscoelastically point-supported orthotropic elastic plate of rectangular shape with the considered locations of the added masses. In the study, for applying the Lagrange equations, the trial function denoting the deflection of the plate is expressed in polynomial form. By using the Lagrange equations, the problem is reduced to the solution of a system of algebraic equations. The influence of the locations of the added masses, mechanical properties characterizing the orthotropy of the plate material and the damping of the supports to the steady state response of the viscoelastically point-supported rectangular plates is investigated numerically for the concentrated load at the centre. Because of the symmetry of the supports and loading condition, only the first three symmetrical modes occur in the considered frequency range. Therefore, only the results of the first three symmetrical modes are given in the present study. Convergence studies are made. The validity of the obtained results is demonstrated by comparing them with other solutions based on the Kirchhoff–Love plate theory.

1. Introduction

Free and forced vibration analysis of plates with added concentrated masses and various boundary conditions is encountered in various engineering applications from printed circuit boards in electronics to the plates used in naval and ocean engineering systems. Therefore, plate

*Corresponding author. Tel.: +90-212-259-70-70/2775; fax: +90-212-236-41-77.
E-mail address: kocaturk@yildiz.edu.tr (T. Kocatürk).

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problems are of great interest to engineers. Also, the present problem is of considerable interest to engineers designing plates with added masses at isolated points. The free vibration analysis of rectangular plates supported at various points with added concentrated masses and based on the Kirchhoff–Love plate theory is investigated and is well known. However, it appears that there is only a limited number of studies on the steady state response of viscoelastically point-supported plates.

A considerable number of publications has been concerned with the free vibration analysis of rectangular isotropic and orthotropic plates supported at various points and based on the Kirchhoff–Love plate theory (for example Refs. [1–8]. Also, there are a lot of studies on the free vibration analysis of plates with added masses: Amba-Rao [9] investigated the vibration of a rectangular plate carrying a concentrated mass. Rossi and Laura [10] investigated the normal modes of a cantilever rectangular plate with a concentrated mass. Cha [11] investigated the free vibration of a rectangular plate carrying a concentrated mass. Wu and Luo [12] investigated the free vibration of a rectangular plate with any number of point masses and translational springs by using a combined analytical and numerical method.

Although there are lots of studies on the free vibration analysis of rectangular plates supported at various points, there is only a limited number of studies on the steady state response of point-supported rectangular plates. The steady state response to a sinusoidally varying force was determined for a viscoelastically point-supported square or rectangular plate by Yamada et al. [13] by using the generalized Galerkin method. A generalization of this study to orthotropic rectangular plates was investigated by Kocatürk [14]. The steady state response to a sinusoidally varying force was determined for a viscoelastically point-supported specially orthotropic square or rectangular plate by Kocatürk and Altíntaş [15] by using an energy-based finite difference method. In the present study, the considered problem is an extension of the study of Kocatürk and Altíntaş [15] to rectangular plates with added point masses at the considered locations by using the Lagrange equations with trial function expressed in terms of a double power series instead of the energy-based finite difference method which was used in Ref. [15]. The effects of the mass ratio and location of the point masses on the changes of the steady state responses are investigated for some considered support parameters by the use of the mentioned numerical solution procedure.

The main purpose of the present work is to analyze the steady state response of a viscoelastically point-supported orthotropic plate with added point masses to a sinusoidally varying force for various values of the mechanical properties characterizing the orthotropy of the plate material by using the Lagrange equations. The considered problems are solved within the framework of the Kirchhoff–Love hypothesis. The convergence study is based on the numerical values obtained for various numbers of polynomial terms. In the numerical examples, the natural frequency parameters and the steady state responses to a sinusoidally varying force are determined for the first three symmetrical mode types. The accuracy of the results is partially established by comparison with previously published accurate results for the corner point supported plates based on the thin plate theory.

2. Analysis

Consider a viscoelastically point-supported rectangular orthotropic plate of side lengths a, b and thickness h with four masses rigidly and symmetrically mounted on the diagonals of the plate under a sinusoidally varying concentrated force $F(t)$ at the centre of the plate as shown in Fig. 1, where k_j is the spring constant (stiffness parameter of the jth support), $P_j(X_{1j}, X_{2j})$ is the support force of a point support at the jth support, M_1, M_2, M_3, M_4 are the masses rigidly mounted on the diagonals of the plate at the co-ordinates $(X_{1M1}, X_{2M1}), (X_{1M2}, X_{2M2}), (X_{1M3}, X_{2M3}), (X_{1M4}, X_{2M4})$, respectively. The axes of the elastic symmetry of the plate material coincide with the OX_1- and OX_2-axis. Therefore, the plate is specially orthotropic. Also, the coordinate axes OX_1 and OX_2 are parallel to the edges of the plate with the origin at O. Although it is possible to take many point supports and masses at arbitrary points, in the numerical investigations here, because of the number of the involved parameters, and in the interests of brevity, it will be considered that the plate is supported symmetrically at the four corner points, where the parameters k_j, c_j are taken to have the same values at all the supports denoted by $k_j = k_s$, $c_j = c_s$ and the points masses are located at the points mentioned above. Also, it is assumed that $M = M_1 = M_2 = M_3 = M_4$. Thus, in the considered loading, locations of the point masses and support conditions, only symmetrical vibrations arise in the plate. Under the above-mentioned conditions, the steady state responses of the viscoelastically corner point-supported plate to a sinusoidally varying force for various orthotropy ratios, mass ratios, locations of masses and damping values will be determined by using the Lagrange equations.

For a plate undergoing sinusoidally varying force $F(t) = Q.\mathrm{e}^{\mathrm{i}\omega t}$, where ω is radian frequency, the strain energy of bending in Cartesian co-ordinates is given by

$$U = \frac{1}{2}\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\left[D_{11}\left(\frac{\partial^2 W}{\partial X_1^2}\right)^2 + 2D_{11}v_{21}\frac{\partial^2 W}{\partial X_1^2}\frac{\partial^2 W}{\partial X_2^2} + D_{22}\left(\frac{\partial^2 W}{\partial X_2^2}\right)^2 + 4D_{66}\left(\frac{\partial^2 W}{\partial X_1 \partial X_2}\right)^2\right]\mathrm{d}X_1\,\mathrm{d}X_2. \tag{1}$$

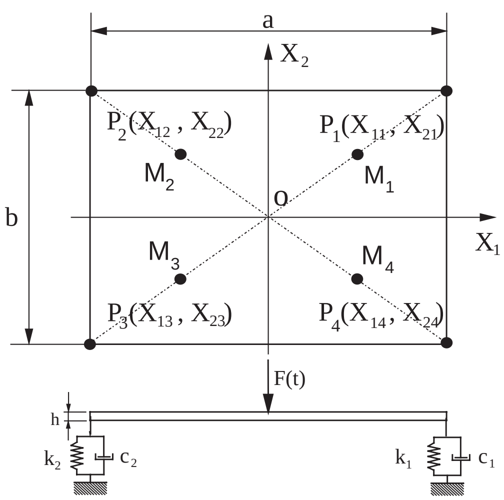

Fig. 1. Viscoelastically point-supported rectangular orthotropic plate with added masses on the two diagonals subjected to an external force.

In Eq. (1), D_{11}, D_{22} and D_{66} are expressed as follows:

$$D_{11} = \frac{E_1 h^3}{12(1 - v_{21}^2/e)}, \quad D_{22} = \frac{E_2 h^3}{12(1 - v_{21}^2/e)}, \quad D_{66} = \frac{G_{12} h^3}{12}, \tag{2}$$

where G_{12} is shear modulus. In deriving the above expressions, the following expressions are used:

$$\frac{v_{12}}{E_1} = \frac{v_{21}}{E_2}, \quad \mathrm{e} = \frac{E_2}{E_1}. \tag{3}$$

Here E_1, E_2 are Young's moduli in the OX_1 and OX_2 directions, respectively, and v_{21} is Poisson's ratio for the strain response in the X_1 direction due to an applied stress in the X_2 direction. The potential energy of the external force is

$$F_e = -F(t)W. \tag{4}$$

With rotary inertia neglected, the kinetic energy of the vibrating plate with four added point masses on the two diagonals of the plate is

$$\begin{aligned} T = &\frac{\rho h \omega^2}{2} \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \dot{W}^2 \, \mathrm{d}X_1 \, \mathrm{d}X_2 \\ &+ \frac{\omega^2 M}{2} [\dot{W}^2(X_{1M1}, X_{2M1}) + \dot{W}^2(X_{1M2}, X_{2M2}) \\ &+ \dot{W}^2(X_{1M3}, X_{2M3}) + \dot{W}^2(X_{1M4}, X_{2M4})], \end{aligned} \tag{5}$$

where ρ is the mass density per unit volume, M is one of the four equal concentrated masses on the diagonal of the plate, and the additive strain energy and dissipation function of viscoelastic supports are

$$\begin{aligned} F_s &= \frac{1}{2} \sum_{i=1}^{4} k_i W_{Si}^2, \\ D &= \frac{1}{2} \sum_{i=1}^{4} c_i (\dot{W}_{Si})^2. \end{aligned} \tag{6}$$

Introducing the following non-dimensional parameters:

$$x_1 = \frac{X_1}{a}, \quad x_2 = \frac{X_2}{b}, \quad \alpha = \frac{a}{b}, \quad \bar{w}(x_1, x_2, t) = W/a, \quad m_t = \frac{4M}{\rho hab}, \tag{7}$$

where m_t is the ratio of the mass of the total concentrated loads to the mass of the plate, the above energy expressions can be written as

$$\begin{aligned} U = &\frac{D_{11}}{2} \int_{-1/2}^{1/2} \int_{-1/2}^{1/2} \left[\frac{1}{\alpha} \left(\frac{\partial^2 \bar{w}}{\partial x_1^2} \right)^2 + 2 v_{21} \, \alpha \frac{\partial^2 \bar{w}}{\partial x_1^2} \frac{\partial^2 \bar{w}}{\partial x_2^2} + e \alpha^3 \left(\frac{\partial^2 \bar{w}}{\partial x_2^2} \right)^2 \right. \\ &\left. + \frac{4 D_{66} \alpha}{D_{11}} \left(\frac{\partial^2 \bar{w}}{\partial x_1^2 \partial x_2^2} \right)^2 \right] \mathrm{d}x_1 \, \mathrm{d}x_2, \end{aligned} \tag{8a}$$

$$T = \frac{a^3 b\rho h\omega^2}{2}\int_{-1/2}^{1/2}\int_{-1/2}^{1/2} \dot{\bar{w}}^2\,\mathrm{d}x_1\,\mathrm{d}x_2 + \tfrac{1}{4}m_t[\dot{\bar{w}}^2(x_{1M1}, x_{2M1}) + \dot{\bar{w}}^2(x_{1M2}, x_{2M2}) + \dot{\bar{w}}^2(x_{1M3}, x_{2M3}) + \dot{\bar{w}}^2(x_{1M4}, x_{2M4})], \tag{8b}$$

$$F_s = \frac{a^2}{2}\sum_{i=1}^{4} k_i \bar{w}_i^2, \quad D = \frac{a^2}{2}\sum_{i=1}^{4} c_i(\dot{\bar{w}}_i)^2, \quad F_e = -aF(t)\bar{w}(0,0). \tag{8c–e}$$

It is known that some expressions satisfying the geometrical boundary conditions are chosen for $\bar{w}(x_1, x_2, t)$ and by using the Lagrange equations, the natural boundary conditions are also satisfied. By using the Lagrange equations, by assuming the displacement $\bar{w}(x_1, x_2, t)$ to be representable by a linear series of admissible functions and adjusting the coefficients in the series to satisfy the Lagrange equations, an approximate solution is found for the displacement function. For applying the Lagrange equations, the trial function $\bar{w}(x_1, x_2, t)$ is approximated by space-dependent polynomial terms $x_1^0, x_1^1, x_1^2, \ldots, x_1^M$ and $x_2^0, x_2^1, x_2^2, \ldots, x_2^N$, and time-dependent generalized displacement co-ordinates $\bar{\mathbf{A}}_{mn}(t)$. Thus

$$\bar{w}(x_1, x_2, t) = \sum_{m=0}^{N}\sum_{n=0}^{N} \bar{\mathbf{A}}_{mn}(t) x_1^m x_2^n, \tag{9}$$

where $\bar{w}(x_1, x_2, t)$ is the steady state response (the transverse deflection) of the plate to a sinusoidally varying force $F(t) = Q.\mathrm{e}^{\mathrm{i}\omega t}$. Each term, x_1^m and x_2^n must satisfy the geometrical boundary conditions. However, in the considered problem, there is no geometrical boundary condition to be satisfied. As it is known, there is no need for these functions to satisfy the natural boundary conditions. However, if the natural boundary conditions were also satisfied when selecting the functions, then the rate of the convergence would be higher.

The function $\bar{w}(x_1, x_2, t)$ that is given by Eq. (9) is substituted in Eqs. (8a–e). Then, application of Lagrange equations yields a set of linear algebraic equations. The Lagrange equations for the considered problem are given as

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{\bar{A}}_{kl}}\right) - \frac{\partial (T-U)}{\partial \bar{A}_{kl}} + \frac{\partial D}{\partial \dot{\bar{A}}_{kl}} + \frac{\partial F_s}{\partial \bar{A}_{kl}} + \frac{\partial F_e}{\partial \bar{A}_{kl}} = 0, \quad k, l = 0, 1, 2, \ldots, N, \tag{10}$$

where the overdot stands for the partial derivative with respect to time. Introducing the following non-dimensional parameters:

$$\kappa_j = \frac{k_j a^3}{bD_{11}}, \quad \gamma_j = \frac{c_j}{\sqrt{\rho h D_{11}}}, \quad \lambda^2 = \frac{\rho h \omega^2 a^4}{D_{11}}, \quad q = \frac{Qa}{D_{11}} \tag{11}$$

and considering that when the force is expressed as $F(t) = Q.\mathrm{e}^{\mathrm{i}\omega t}$, then the time-dependent generalized functions can be expressed as follows:

$$\bar{\mathbf{A}}_{mn}(t) = \mathbf{A}_{mn}\mathrm{e}^{\mathrm{i}\omega t}. \tag{12}$$

In Eq. (12), $\mathbf{A}_m$ is a complex variable containing a phase angle. Dimensionless complex amplitude of the displacement of a point of the plate can be expressed as

$$w(x_1, x_2) = \sum_{m=0}^{M} \sum_{n=0}^{N} \mathbf{A}_{mn} x_1^m x_2^n. \tag{13}$$

By using Eq. (10), the following set of linear algebraic equations is obtained which can be expressed in the following matrix form:

$$[\mathbf{A}]\{\mathbf{A}_{mn}\} + \mathrm{i}\lambda\gamma[\mathbf{B}]\{\mathbf{A}_{mn}\} - \lambda^2[\mathbf{C}]\{\mathbf{A}_{mn}\} = \{\mathbf{q}\}, \tag{14}$$

where [**A**], [**B**] and [**C**] are coefficient matrices obtained by using Eq. (10).

For free vibration analysis, when the external force and damping of the supports are zero in Eq. (14), this situation results in a set of linear homogeneous equations that can be expressed in the following matrix form:

$$[\mathbf{A}]\{\mathbf{A}_{mn}\} - \lambda^2[\mathbf{C}]\{\mathbf{A}_{mn}\} = \{\mathbf{0}\}. \tag{15}$$

By increasing the polynomial terms, the accuracy can be increased. The maximum total magnitude of the reaction forces of the supports is given by

$$\sum_{j=1}^{4} P_j = \sum_{j=1}^{4} (k_j + \mathrm{i}c_j\omega)a \sum_{m=0}^{N} \sum_{n=0}^{N} \mathbf{A}_{mn} x_{1j}^m x_{2j}^n \tag{16}$$

and therefore the force transmissibility at the supports is determined by

$$T_R = \sum_{j=1}^{4} P_j/Q = \sum_{j=1}^{4} (\kappa_j + \mathrm{i}\gamma_j\lambda) \sum_{m=0}^{M} \sum_{n=0}^{N} \mathbf{A}_{mn} x_{1j}^m x_{2j}^n/(\alpha q). \tag{17}$$

The number of unknown coefficients is $(N+1)(N+1)$. Again, the number of equations that can be written for each $\mathbf{A}_{mn}$ coefficient by using Eq. (10) is $(N+1)(N+1)$, which is given in matrix form by Eq. (14). Therefore, the total number of these equations is equivalent to the total number of unknown displacements and these unknowns can be determined by using the above mentioned equations.

The eigenvalues (characteristic values) λ are found from the condition that the determinant of the system of equations given by Eq. (15) must vanish.

3. Numerical results

The steady state response to a point force $F(t)$ acting at the centre of an orthotropic square plate with four added concentrated masses attached on the two diagonals of the plate, viscoelastically supported at four points which are symmetrically located at the corners, is calculated numerically. The parameters κ_i and γ_i are taken as having the same respective values at all the supports denoted by $\kappa_i = \kappa_s$ and $\gamma_i = \gamma_s$. Because of the structural symmetry and symmetry of the external force, only symmetrical vibrations arise in the plate. The symbol *SS* represents symmetrical vibration with respect to centre-lines.

Table 1
Comparison of the obtained results with the existing results and convergence study of frequency parameters λ for corner point supported square plates

	Determinant size	SS-1	SS-2	SS-3
(a) $\nu_{21} = 0.3$, $\alpha = 1$, $e = 1$				
Present study $\kappa = 1 \times 10^8$	9×9	7.11180	19.72570	45.55991
	16×16	7.11093	19.59627	44.37619
	25×25	7.11089	19.59614	44.36968
	36×36	7.11088	19.59614	44.36961
Narita [4]	37×37	7.11089	19.5961	44.3696
Kocatürk and İlhan [8]	36×36	7.11088	19.5961	44.3696
Venkateswara Rao et al. [5]	180×180	7.11088	19.5961	—
Kerstens [1]	36×36	7.15	19.49	43.89
(b) $\nu_{21} = 0.333$, $\alpha = 1$, $e = 1$				
Present study $\kappa = 1 \times 10^8$	9×9	7.10274	19.34726	45.19786
	16×16	7.10196	19.22399	44.04902
	25×25	7.10192	19.22386	44.04289
	36×36	7.10192	19.22386	44.04282
Narita [4]	37×37	7.10192	19.2239	44.0428
Kocatürk and İlhan [8]	36×36	7.10192	19.22386	44.04282
Gorman [3]		7.18	19.22	44.28

Although there are no existing results on the forced vibration analysis of the considered problem, there are many existing results on the free vibration analysis of corner point-supported isotropic plates. Therefore, to show the accuracy of the solution procedure, a short investigation of the free vibration of an elastically point-supported plate is made only to compare the obtained results with the existing results of free vibration analysis of elastically point-supported plates: The natural frequencies of the elastically point-supported plate are determined by calculating the eigenvalues λ of the frequency Eq. (15). The force transmissibilities are determined for various damping parameters γ_s for various values of mass ratios m_t and stiffness parameters κ_s by using Eqs. (14, 17). In all of the numerical calculations, ν_{21} is taken as 0.3 and the locations of the point supports are chosen at the corners of the plate.

In the frequency and steady state response equations, m and n are odd or even integers depending on the vibration mode. For example, the SA mode is symmetric about the x_2-axis and antisymmetric about the x_1-axis. Therefore, for the SA mode, $m = 0, 2, 4, \ldots$ and $n = 1, 3, 5, \ldots$. However, in the present study, because of the symmetry of the problem, only the SS modes are dealt with. Therefore, for the SS modes, $m = 0, 2, 4, \ldots$ and $n = 0, 2, 4, \ldots$. In the numerical calculations, G_{12} is taken as given by Szilard [16] as follows:

$$G_{12} \approx \frac{E_1 \sqrt{e}}{2(1 + \nu_{21}\sqrt{1/e})}. \tag{18}$$

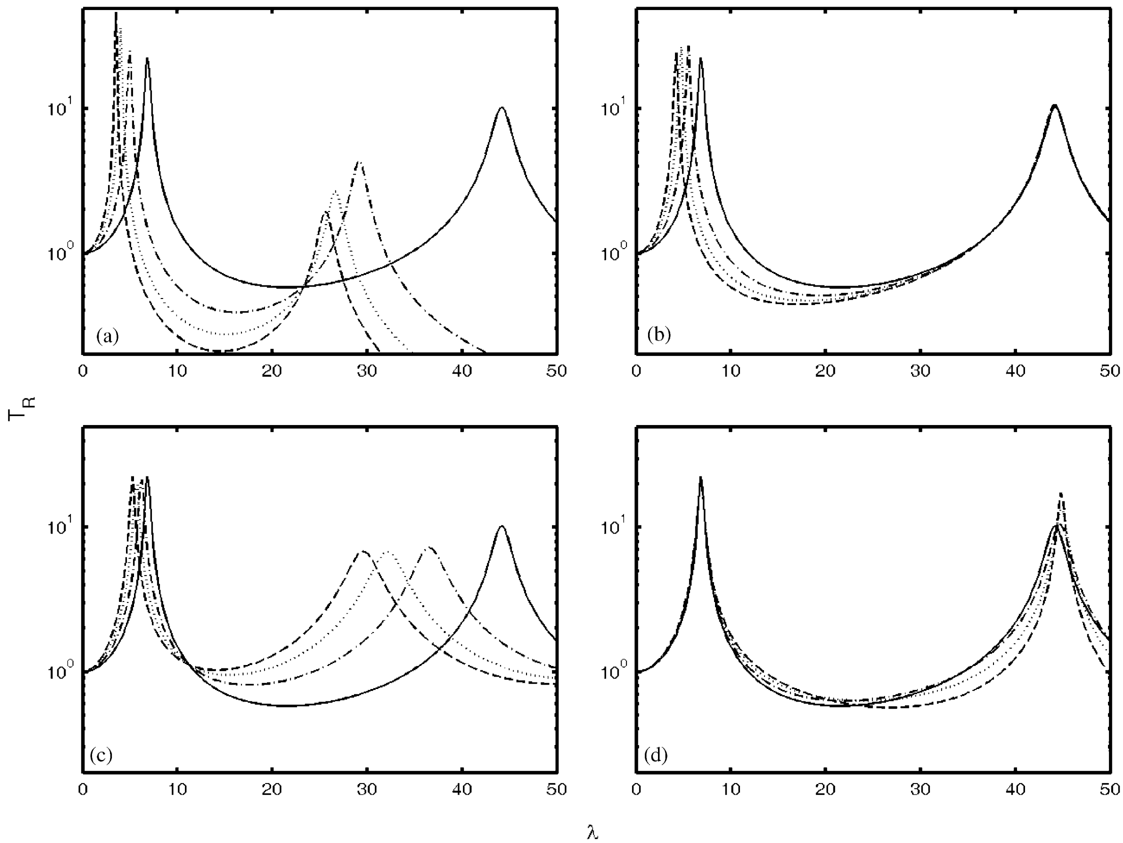

Fig. 2. The force transmissibilities for various values of m_t for $E_2/E_1 = 1$ for (a) $x_M = 0$, (b) $x_M = 0.25$, (c) $x_M = 0.375$, (d) $x_M = 0.5$. $\kappa_s = 100$, $\gamma_s = 10$. $m_t = 0$ –, $m_t = 0.5$ -··-, $m_t = 1.0$ ···, $m_t = 1.5$ - -.

It is possible to simulate infinite lateral support stiffness by setting the translational stiffness coefficient equal to 1×10^8 at all the supports to compare the obtained results with the existing results of the point supported plates. Also, by setting the translational stiffness coefficient equal to zero at all the supports, a completely free plate situation can be obtained. In the case when the mass ratio is zero, then the considered problem is reduced to the problem investigated by Kocatürk and Altíntaş [15] by using the energy-based finite difference method. The results obtained are compared in Table 1 with those of Kerstens [1], Gorman [3], Narita [4], Venkateswara Rao et al. [5] and Kocatürk and İlhan [8] for the SS-1, SS-2, SS-3 natural frequencies of an isotropic square plate supported at the corners for $\nu_{21} = 0.3$. Also, the convergence is tested in Table 1 by taking the number of terms $(N+1)(N+1) = 3 \times 3, 4 \times 4, 5 \times 5, 6 \times 6$. The corresponding determinant size becomes $9 \times 9, 16 \times 16, 25 \times 25, 36 \times 36$, respectively. The exactness of the present results with the results of Narita [4], Kocatürk and İlhan [8] is because all of the studies use the polynomial terms for the trial function and in the case of zero damping, the present problem is reduced to those of Narita [4] and Kocatürk and İlhan [8]:

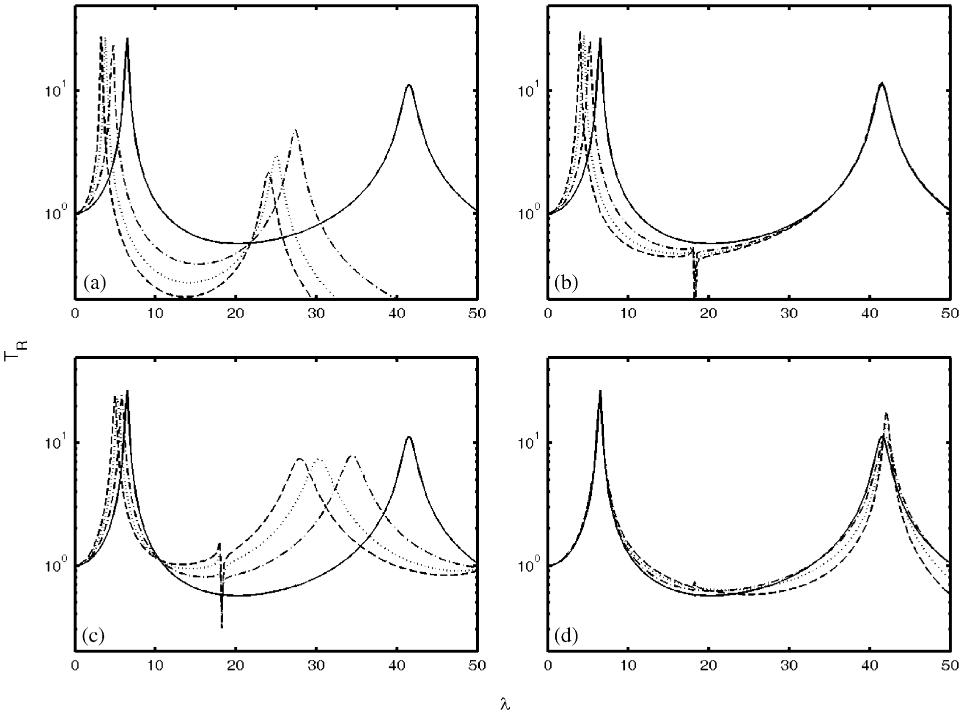

Fig. 3. The force transmissibilities for various values of m_t for $E_2/E_1 = 0.8$ for (a) $x_M = 0$, (b) $x_M = 0.25$, (c) $x_M = 0.375$, (d) $x_M = 0.5$. $\kappa_s = 100$, $\gamma_s = 10$. $m_t = 0$ –, $m_t = 0.5$ -·-, $m_t = 1.0$ ···, $m_t = 1.5$ - -.

Namely, in the case of zero damping, the present method is reduced to the classical Ritz method. For determining the mode shapes of the vibration, for the considered eigenvalue, a coefficient is taken as known in Eq. (15), then the other coefficients are determined according to this known coefficient. After that, by using Eq. (13), the mode shape of the considered vibration can be determined. It is seen from Table 1 that the present converged values for plates without added masses show very good agreement with the existing results [1,3–5,8]. As it is observed from Table 1, the frequency parameter decreases as the number of the polynomial terms increases. It means that the convergence is from above. By increasing the number of the polynomial terms, the exact value can be approached from above. It should be remembered that energy methods always overestimate the fundamental frequency, so with more refined analyses, the exact value can be approached from above. The convergence study indicates that the calculated values are converged to within five significant figures.

From here on, in the calculation of the results of the present study, 4×4 terms of the polynomial series are used, namely the size of the determinant is 16×16. It was determined by previous studies [13–15] that, when increasing the stiffness parameter κ_s, the frequency parameters increase monotonically and ultimately become the values of a simply point-supported plate. Figs. 2–4 show the force transmissibilities for various values of m_t for $E_2/E_1 = 1$, $E_2/E_1 = 0.8$, $E_2/E_1 = 0.6$ for $x_M = 0, 0.25, 0.375, 0.5$, respectively. In Tables 2–4 the frequencies at which the

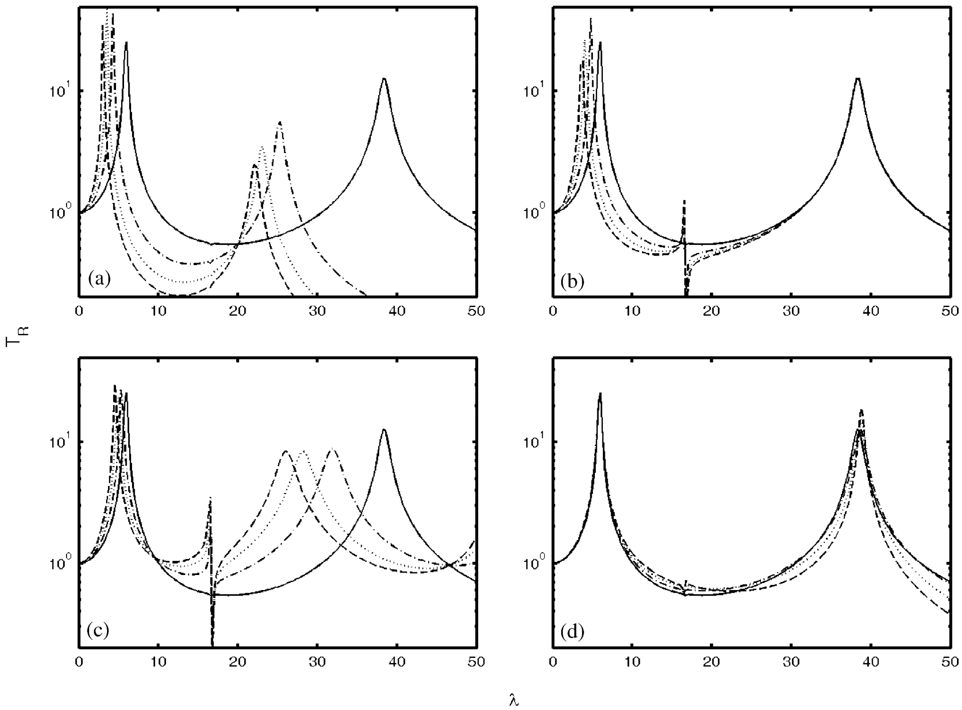

Fig. 4. The force transmissibilities for various values of m_t for $E_2/E_1 = 0.6$ for (a) $x_M = 0$, (b) $x_M = 0.25$, (c) $x_M = 0.375$, (d) $x_M = 0.5$. $\kappa_s = 100$, $\gamma_s = 10$. $m_t = 0$ –, $m_t = 0.5$ -·-, $m_t = 1.0$ ···, $m_t = 1.5$ - -.

peak values of the force transmissibilities occur are determined for various damping parameters γ_s for $m_t=0.0,\ 0.5,\ 1.0,\ 1.5$, $x_M = 0.0,\ 0.25,\ 0.375,\ 0.5$ by using Eqs. (13, 14, 16, 17) for $E_2/E_1 = 1$, $E_2/E_1 = 0.8$, $E_2/E_1 = 0.6$, respectively. In the isotropic case, in the SS-2 vibration mode, for $E_2/E_1 = 1$, the frequency parameter remains constant without being affected by the variation of κ_s [13–15] and by the variation of the mass ratio m_t when the masses are on the two diagonals. Also, the SS-2 mode does not occur for the related forced vibration of the viscoelastically point-supported plate with added concentrated masses on the two diagonals in the isotropic case for the considered parameters as seen from Fig. 2. This is because the two diagonals of the plate coincide with the nodal lines for the SS-2 mode. When the value of E_2/E_1 is unity, the SS-2 vibration with nodal lines coinciding with the two diagonals does not arise in the plate. Therefore, in the case of SS-2 mode, for the isotropic case, the line for this related mode in Table 3 is not shown. However, in the orthotropic case, in the SS-2 vibration mode, frequency parameters change with the variation of m_t and κ_s which can be seen from Figs. 3 and 4. It means that the nodal lines do not coincide with the two diagonals of the plate when the plate is orthotropic. It is deduced from Tables 2–4 and Figs. 5–7 that, for SS-1 mode, for constant m_t value, for increasing values of x_M, the frequency parameters increase. However, for SS-3 mode, for constant m_t value, for increasing values of x_M, the frequency parameters increase until a certain value of x_M. After that certain value, the frequency values decrease. However, the frequencies cannot be greater than the

Table 2
The frequencies and dimensionless damping coefficients at which the peak values of the force transmissibilities occur: $v_{21} = 0.3, \kappa_s = 100, e = 1$

Modes	m_t	$\gamma_s = 0$	$\gamma_s = 1$	$\gamma_s = 3$	$\gamma_s = 5$	$\gamma_s = 10$	$\gamma_s = 20$	$\gamma_s = 2000$
$x_{1M} = x_{2M} = 0$, $e = 1$								
SS-1	0	6.72	6.72	6.74	6.76	6.83	6.97	7.11
	0.5	4.88	4.89	4.89	4.90	4.92	4.99	5.10
	1	4.01	4.01	4.01	4.01	4.03	4.07	4.17
	1.5	3.48	3.48	3.48	3.48	3.49	3.52	3.61
SS-3	0	40.62	41.03	42.77	43.64	44.17	44.32	44.38
	0.5	27.42	27.51	28.09	28.62	29.14	29.33	29.40
	1	25.07	25.15	25.62	26.10	26.63	26.83	26.91
	1.5	24.11	24.17	24.60	25.06	25.60	25.81	25.89
$x_{1M} = x_{2M} = 0.25$, $e = 1$								
SS-1	0	6.72	6.72	6.74	6.76	6.83	6.97	7.11
	0.5	5.41	5.42	5.42	5.43	5.48	5.57	5.72
	1	4.66	4.66	4.66	4.67	4.70	4.77	4.92
	1.5	4.15	4.15	4.15	4.15	4.18	4.24	4.38
SS-3	0	40.62	41.03	42.77	43.64	44.17	44.32	44.38
	0.5	40.62	41.02	42.76	43.62	44.15	44.29	44.35
	1	40.61	41.02	42.75	43.61	44.13	44.28	44.33
	1.5	40.61	41.02	42.75	43.60	44.12	44.27	44.32
$x_{1M} = x_{2M} = 0.375$, $e = 1$								
SS-1	0	6.72	6.72	6.74	6.76	6.83	6.97	7.11
	0.5	6.06	6.06	6.07	6.09	6.16	6.31	6.49
	1	5.55	5.55	5.56	5.58	5.64	5.78	6.00
	1.5	5.14	5.14	5.15	5.16	5.22	5.36	5.60
SS-3	0	40.62	41.03	42.77	43.64	44.17	44.32	44.38
	0.5	32.21	32.44	33.97	35.37	36.53	36.88	37.01
	1	28.34	28.49	29.54	30.79	32.12	32.58	32.75
	1.5	26.25	26.36	27.16	28.23	29.57	30.08	30.27
$x_{1M} = x_{2M} = 0.5$, $e = 1$								
SS-1	0	6.72	6.72	6.74	6.76	6.83	6.97	7.11
	0.5	6.70	6.70	6.72	6.74	6.83	6.97	7.11
	1	6.68	6.68	6.70	6.72	6.82	6.97	7.11
	1.5	6.65	6.66	6.67	6.70	6.81	6.98	7.11
SS-3	0	40.62	41.03	42.77	43.64	44.17	44.32	44.38
	0.5	27.51	27.52	—	—	—	—	44.38
	1	20.48	20.29	18.49	—	—	—	44.38
	1.5	17.02	16.86	15.17	—	—	—	44.38

frequencies of the plate without added point masses. When the γ_s values or κ_s values are big enough, then, for $x_M = 0.5$, there is no effect of m_t on the frequency parameters and on the force transmissibilities. When $x_M = 0.0$, then the mass ratio is the most effective on SS-1 mode frequencies. Also, when $x_M = 0.5$, then the mass ratio is the most effective on SS-3 mode

Table 3
The frequencies and dimensionless damping coefficients at which the peak values of the force transmissibilities occur: $v_{21} = 0.3, \kappa_s = 100, e = 0.8$

Modes	m_t	$\gamma_s = 0$	$\gamma_s = 1$	$\gamma_s = 3$	$\gamma_s = 5$	$\gamma_s = 10$	$\gamma_s = 20$	$\gamma_s = 2000$
$x_{1M} = x_{2M} = 0$, $e = 0.8$								
SS-1	0	6.36	6.36	6.37	6.39	6.45	6.56	6.69
	0.5	4.63	4.63	4.63	4.64	4.66	4.71	4.81
	1	3.80	3.80	3.80	3.80	3.81	3.85	3.94
	1.5	3.30	3.30	3.30	3.30	3.31	3.33	3.41
SS-2	0	18.20	18.20	18.20	18.19	18.33	18.22	18.22
	0.5	18.19	18.19	18.20	18.20	18.20	18.21	18.21
	1	18.19	18.19	18.19	18.20	18.20	18.20	18.21
	1.5	18.19	18.19	18.19	18.20	18.20	18.22	18.22
SS-3	0	38.66	38.97	40.32	41.04	41.50	41.63	41.68
	0.5	25.98	26.06	26.50	26.92	27.36	27.52	27.58
	1	23.75	23.80	24.16	24.54	24.98	25.16	25.23
	1.5	22.82	22.88	23.20	23.57	24.01	24.20	24.27
$x_{1M} = x_{2M} = 0.25$, $e = 0.8$								
SS-1	0	6.36	6.36	6.37	6.39	6.45	6.56	6.69
	0.5	5.13	5.13	5.13	5.14	5.18	5.26	5.39
	1	4.41	4.41	4.41	4.42	4.44	4.50	4.63
	1.5	3.93	3.93	3.93	3.93	3.95	4.00	4.13
SS-2	0	18.20	18.20	18.20	18.19	18.33	18.22	18.22
	0.5	18.20	18.20	18.20	18.20	18.19	18.15	18.22
	1	18.20	18.20	18.20	18.20	18.20	18.21	18.21
	1.5	18.20	18.20	18.20	18.20	18.20	18.21	18.21
SS-3	0	38.66	38.97	40.32	41.04	41.50	41.63	41.68
	0.5	38.65	38.96	40.32	41.03	41.48	41.61	41.66
	1	38.65	38.96	40.31	41.02	41.47	41.60	41.65
	1.5	38.65	38.96	40.31	41.01	41.46	41.59	41.64
$x_{1M} = x_{2M} = 0.375$, $e = 0.8$								
SS-1	0	6.36	6.36	6.37	6.39	6.45	6.56	6.69
	0.5	5.74	5.74	5.75	5.77	5.82	5.94	6.10
	1	5.26	5.26	5.27	5.28	5.33	5.45	5.64
	1.5	4.88	4.88	4.89	4.90	4.94	5.05	5.26
SS-2	0	18.20	18.20	18.20	18.19	18.33	18.22	18.22
	0.5	18.18	18.18	18.18	18.19	18.19	18.20	18.20
	1	18.16	18.16	18.17	18.17	18.18	18.19	18.19
	1.5	18.14	18.14	18.15	18.16	18.17	18.18	18.18
SS-3	0	38.66	38.97	40.32	41.04	41.50	41.63	41.68
	0.5	30.84	31.04	32.28	33.42	34.41	34.73	34.84
	1	27.18	27.30	28.17	29.19	30.31	30.72	30.87
	1.5	25.19	25.28	25.95	26.83	27.95	28.40	28.57
$x_{1M} = x_{2M} = 0.5$, $e = 0.8$								
SS-1	0	6.36	6.36	6.37	6.39	6.45	6.56	6.69
	0.5	6.35	6.35	6.36	6.38	6.44	6.56	6.69
	1	6.33	6.33	6.34	6.36	6.44	6.56	6.69

Table 3 (*continued*)

Modes	m_t	$\gamma_s = 0$	$\gamma_s = 1$	$\gamma_s = 3$	$\gamma_s = 5$	$\gamma_s = 10$	$\gamma_s = 20$	$\gamma_s = 2000$
	1.5	6.31	6.31	6.32	6.35	6.43	6.57	6.69
SS-2	0	18.20	18.20	18.20	18.19	18.33	18.22	18.22
	0.5	18.19	18.19	18.18	18.17	18.26	18.22	18.22
	1	18.14	18.14	18.10	18.46	18.24	18.22	18.22
	1.5	16.81	16.67	15.23	18.26	18.23	18.22	18.22
SS-3	0	38.66	38.97	40.32	41.04	41.50	41.63	41.68
	0.5	27.31	27.45	—	—	—	—	41.66
	1	20.41	20.26	19.15	—	—	—	41.65
	1.5	18.30	18.30	18.28	—	—	—	41.64

frequencies. In the case of over-large values of κ_s or γ_s, the plate behaves like a simply point-supported plate. The resonant peaks occur at different values of λ while changing the damping parameter γ_s between 0 and ∞. However, the frequency parameter λ remains between the frequency parameters λ obtained for $\gamma_s = 0$ and ∞ which was mentioned in Ref. [15].

It is seen from Figs. 3, 4, 6 and 7 that the effect of the values and locations of the masses is insignificant on the value of frequency parameter λ of the *SS*-2 mode in the case of orthotropic plates for the considered orthotrophy ratios. Also, it can be deduced from Table 2 and Figs. 5–7 that, within some values of damping parameters, the peak values of the force transmissibilities become minimal. In some mass ratios and damping parameters, for $x_M = 0.5$, $e = 0.8$, the peak values of the force transmissibilities does not occur for *SS*-3 mode range. A similar situation is valid for $x_M = 0.5$, $e = 0.6$, for the force transmissibilities of *SS*-3 mode range. When $x_M = 0.25$, for all orthotropy values and damping parameters, it can be said that there is almost no effect of the mass ratios on the frequency values of the *SS*-2 and *SS*-3 modes. It can be seen from these figures that with the variation of damping parameter γ_s, an optimum damping parameter can be obtained for which the values of the resonant peaks are minimal. This situation was investigated before by Kocatürk and Altíntaş [15] for viscoelastically point-supported plates without masses. The same situation is valid in the present study. However, for the conciseness of the study, the optimum values of the damping parameters are not investigated here.

As far as the authors know, there are no published results to compare the present obtained results of viscoelastically point-supported orthotropic plates with added concentrated masses on the diagonals of the plate.

4. Conclusions

By using the Lagrange equations, the natural frequencies for the *SS* mode of elastically point-supported orthotropic rectangular plates with added concentrated masses and the steady state response of a viscoelastically point-supported orthotropic square plates again with added concentrated masses to a sinusoidally varying force has been studied and compared with the existing results. To use the Lagrange equations with the trial function in the polynomial form is a very good way for studying the structural behaviour of plates elastically or viscoelastically

Table 4
The frequencies and dimensionless damping coefficients at which the peak values of the force transmissibilities occur: $v_{21} = 0.3, \kappa_s = 100,\ e = 0.6$

Modes	m_t	$\gamma_s = 0$	$\gamma_s = 1$	$\gamma_s = 3$	$\gamma_s = 5$	$\gamma_s = 10$	$\gamma_s = 20$	$\gamma_s = 2000$
$x_{1M} = x_{2M} = 0,\ e = 0.6$								
SS-1	0	5.84	5.84	5.85	5.86	5.90	5.98	6.09
	0.5	4.25	4.25	4.26	4.26	4.27	4.31	4.40
	1	3.50	3.50	3.50	3.50	3.51	3.53	3.60
	1.5	3.03	3.03	3.03	3.04	3.04	3.06	3.12
SS-2	0	16.58	16.58	16.57	16.56	16.95	16.67	16.65
	0.5	16.58	16.58	16.57	16.55	16.31	16.67	16.65
	1	16.58	16.57	16.57	16.55	16.37	16.67	16.65
	1.5	16.57	16.57	16.57	16.55	16.39	16.67	16.65
SS-3	0	36.28	36.49	37.44	37.98	38.35	38.46	38.50
	0.5	24.25	24.30	24.59	24.89	25.23	25.36	25.41
	1	22.14	22.17	22.41	22.68	23.01	23.16	23.22
	1.5	21.26	21.30	21.52	21.77	22.10	22.25	22.32
$x_{1M} = x_{2M} = 0,\ e = 0.6$								
SS-1	0	5.84	5.84	5.85	5.86	5.90	5.98	6.09
	0.5	4.71	4.71	4.72	4.72	4.74	4.80	4.91
	1	4.06	4.06	4.06	4.06	4.08	4.12	4.23
	1.5	3.61	3.61	3.62	3.62	3.63	3.66	3.77
SS-2	0	16.58	16.58	16.57	16.56	16.95	16.67	16.65
	0.5	16.56	16.56	16.56	16.57	16.59	16.61	16.62
	1	16.55	16.55	16.56	16.57	16.58	16.60	16.61
	1.5	16.54	16.54	16.55	16.56	16.58	16.59	16.60
SS-3	0	36.28	36.49	37.44	37.98	38.35	38.46	38.50
	0.5	36.27	36.49	37.44	37.97	38.34	38.45	38.49
	1	36.27	36.49	37.43	37.97	38.33	38.44	38.48
	1.5	36.27	36.49	37.43	37.96	38.33	38.44	38.48
$x_{1M} = x_{2M} = 0,\ e = 0.6$								
SS-1	0	5.84	5.84	5.85	5.86	5.90	5.98	6.09
	0.5	5.28	5.28	5.29	5.30	5.33	5.42	5.55
	1	4.85	4.85	4.85	4.86	4.89	4.98	5.13
	1.5	4.50	4.50	4.50	4.51	4.54	4.62	4.79
SS-2	0	16.58	16.58	16.57	16.56	16.95	16.67	16.65
	0.5	16.50	16.50	16.51	16.52	16.55	16.57	16.59
	1	16.42	16.42	16.43	16.49	16.49	16.52	16.54
	1.5	16.35	16.35	16.37	16.39	16.44	16.48	16.50
SS-3	0	36.28	36.49	37.44	37.98	38.35	38.46	38.50
	0.5	29.17	29.32	30.23	31.09	31.87	32.13	32.23
	1	25.78	25.87	26.52	27.28	28.15	28.48	28.60
	1.5	23.95	24.02	24.52	25.16	26.02	26.38	26.51
$x_{1M} = x_{2M} = 0,\ e = 0.6$								
SS-1	0	5.84	5.84	5.85	5.86	5.90	5.98	6.09
	0.5	5.83	5.83	5.84	5.85	5.89	5.98	6.09
	1	5.82	5.82	5.83	5.84	5.89	5.98	6.09

Table 4 (*continued*)

Modes	m_t	$\gamma_s=0$	$\gamma_s=1$	$\gamma_s=3$	$\gamma_s=5$	$\gamma_s=10$	$\gamma_s=20$	$\gamma_s=2000$
	1.5	5.81	5.81	5.81	5.83	5.88	5.98	6.09
SS-2	0	16.58	16.58	16.57	16.56	16.95	16.67	16.65
	0.5	16.55	16.55	16.53	16.49	16.86	16.67	16.65
	1	16.46	16.46	16.41	—	16.78	16.67	16.65
	1.5	15.97	15.92	15.18	—	16.74	16.67	16.65
SS-3	0	36.28	36.49	37.44	37.98	38.35	38.46	38.50
	0.5	27.12	27.47	—	—	—	—	38.50
	1	20.37	20.26	19.45	17.83	—	—	38.50
	1.5	17.42	17.38	17.14	16.94	—	—	38.50

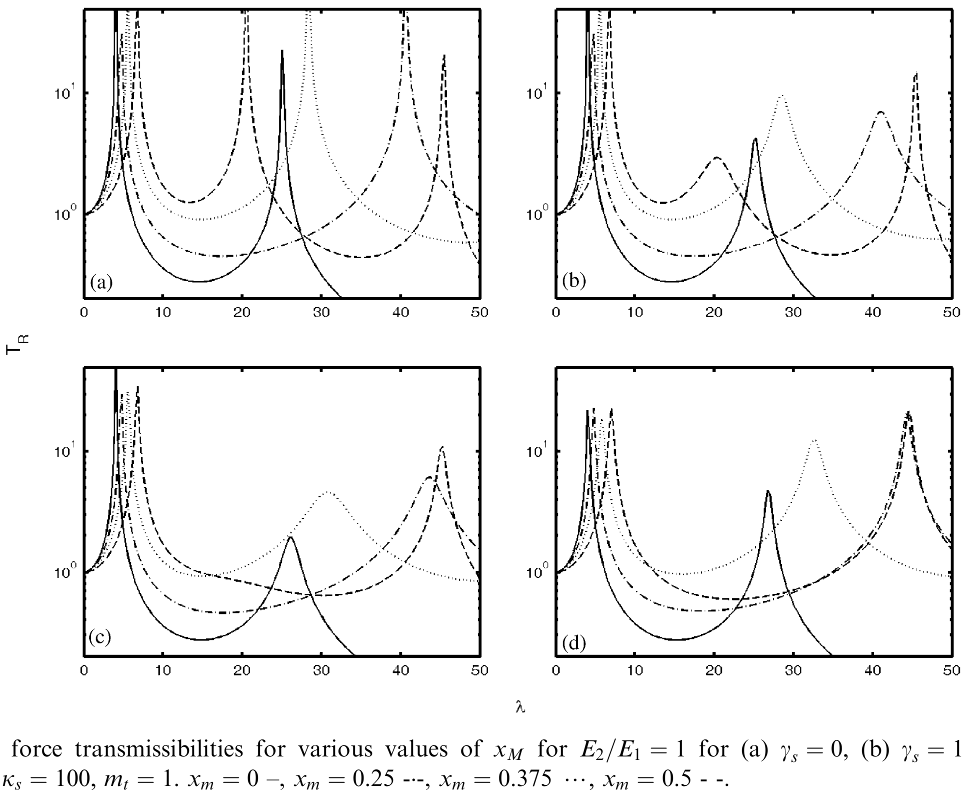

Fig. 5. The force transmissibilities for various values of x_M for $E_2/E_1=1$ for (a) $\gamma_s=0$, (b) $\gamma_s=1$, (c) $\gamma_s=5$, (d) $\gamma_s=20$. $\kappa_s=100$, $m_t=1$. $x_m=0$ –, $x_m=0.25$ -·-, $x_m=0.375$ ···, $x_m=0.5$ - -.

point-supported. For the same accuracy level, it needs considerably fewer degrees of freedom than the finite element method and energy-based finite difference method.

By the application of the above-mentioned solution technique, for the *SS* vibration mode family, the first three values of the natural frequencies are determined, the convergence characteristics of the frequency parameters are investigated numerically for orthotropic square

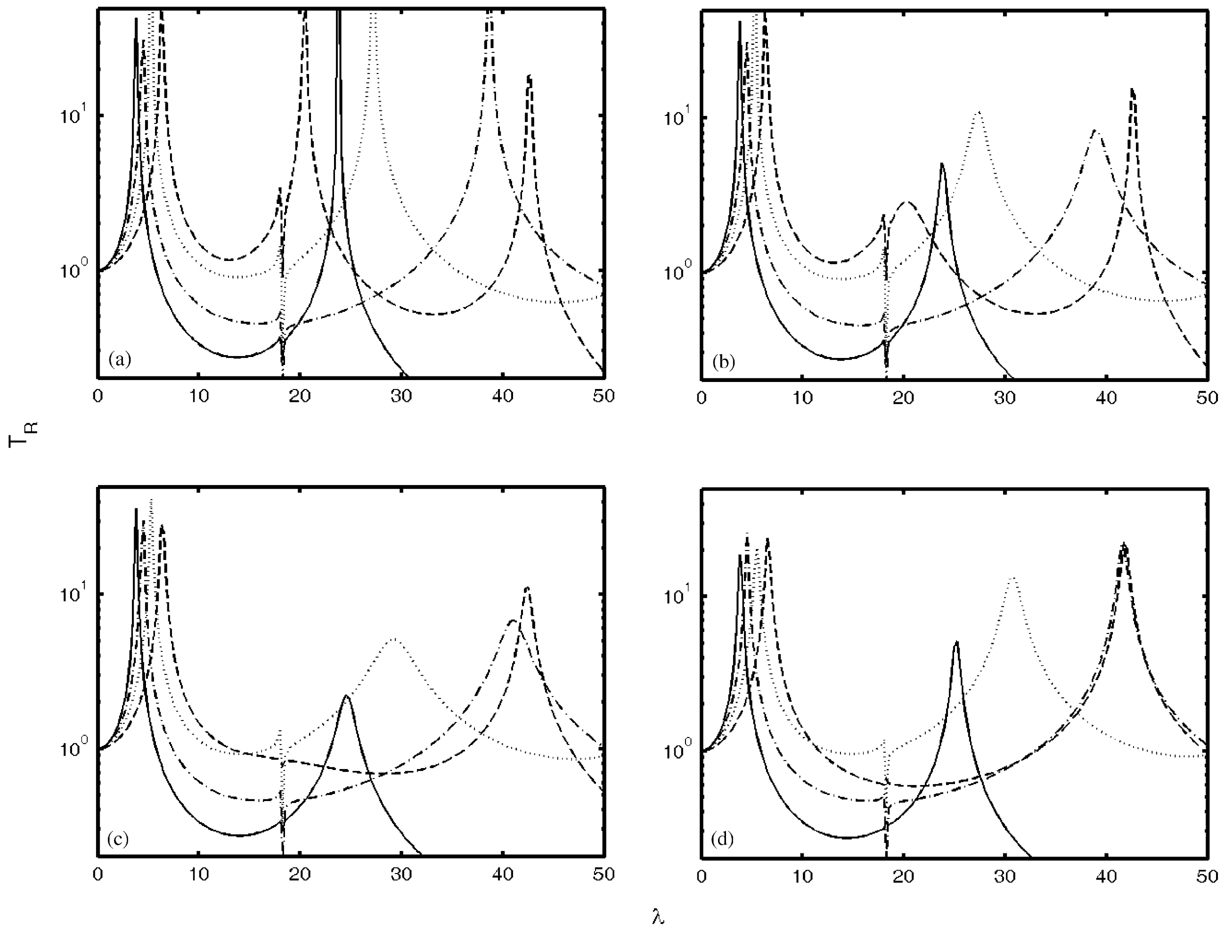

Fig. 6. The force transmissibilities for various values of x_M for $E_2/E_1 = 0.8$ for (a) $\gamma_s = 0$, (b) $\gamma_s = 1$, (c) $\gamma_s = 5$, (d) $\gamma_s = 20$. $\kappa_s = 100$, $m_t = 1$. $x_m = 0$ –, $x_m = 0.25$ -·-, $x_m = 0.375$ ···, $x_m = 0.5$ - -.

plates with added concentrated masses on the two diagonals of the plate elastically supported at four points at the corners and compared with the existing results. It is seen that the rate of convergence is very high. The effect of the mass ratios, the locations of point masses and orthotropy on the frequency parameters is investigated and shown in Tables 2–4.

The response curves to a sinusoidally varying point force acting at the centre are determined numerically for orthotropic square plates with point masses, viscoelastically supported at four points at the corners. The effect of the mass ratio, locations of the supports, orthotropy and damping of the supports on the response curves is investigated and shown in Figs. 2–7 and Tables 2–4. It is seen that, because of the orthotropy, the *SS*-2 mode occurs in the plate.

All of the obtained results are very accurate and may be useful for designing mechanical systems under external dynamic loads.

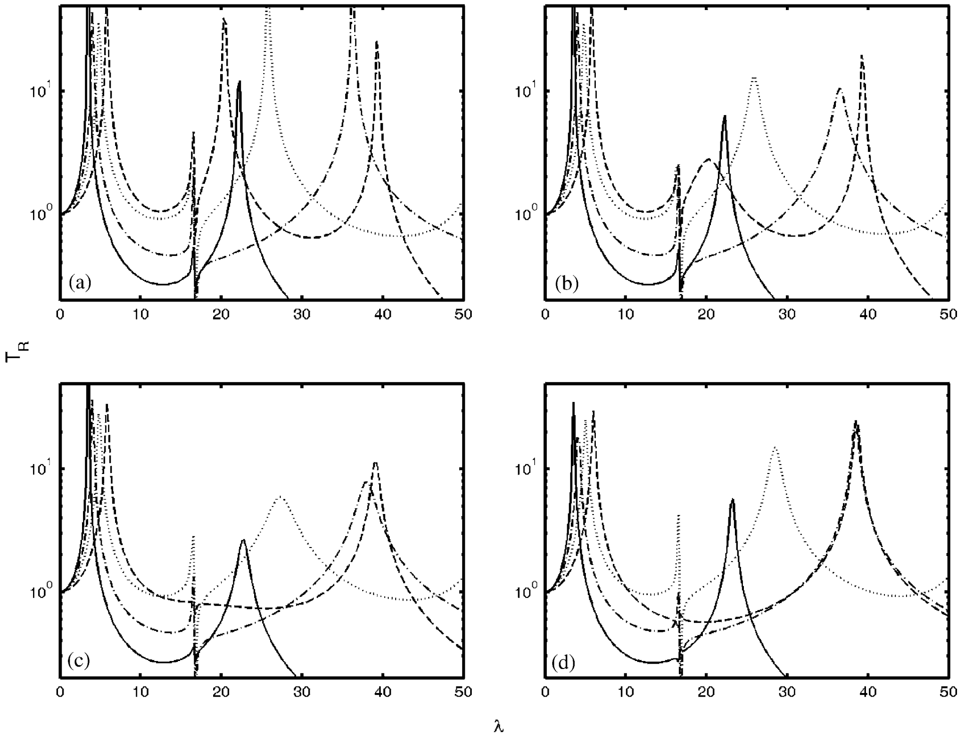

Fig. 7. The force transmissibilities for various values of x_M for $E_2/E_1 = 0.6$ for (a) $\gamma_s = 0$, (b) $\gamma_s = 1$, (c) $\gamma_s = 5$, (d) $\gamma_s = 20$. $\kappa_s = 100$, $m_t = 1$. $x_m = 0$ –, $x_m = 0.25$ -·-, $x_m = 0.375$ ···, $x_m = 0.5$ - -.

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